理 學 院
109 學年度第一學期模組化課程

電腦圖像背後的線性代數
Linear Algebra behind Computer Graphics

授課教師	任職單位	畢業學校
許瑞麟	國立成功大學數學系	北卡羅來納州立大學

課程類別	學分數	選必修	開課人數	其他注意事項
Lecture + Recitation	1	選修	50	無

先修課程或先備能力

無

建議修課年級

不設限

建議修課學生背景

適合各領域學生修習

教學方法

講授 70%，練習 30%

評量方式

問題考試 40%，期末報告 40%，發問問題 20%

補充說明：

(i) 星期二、三、四、五每天 9:00-9:50 先小考，考試內容為前一天上課內容。

(ii) 考試目的是為了提示課程重點，並確保學習成效。考試方式為開放式引導式考試。學生可於考試時舉手發問，老師或助教會趨前給予適當提示並引導作答。不舉手發問視為自動放棄權利。

(iii) 每次小考佔 10%，四次共 40%。

(iv) 期末報告主要是撰寫課程 1000 字(以上)心得報告，內容必須包含「課程總結」、「課程內容自我整理筆記」、以及「學習心得」三部分。於課程結束後隔周一，繳交 pdf 檔上傳至 NCKU Moodle system。

(v) 發問問題佔 20 分。學生可於上課發問問題；課間或課後向在場助教、老師發問問題；或與助教、老師約時間至其辦公室問問題。問題可以是課程相關，也可以是其他疑問的請教。發問問題將會有紀錄，於課程結束後結算並計算成績。

學習規範

無

課程概述

無疑地，線性代數是一個具有海量內容的科目。對大多數的學生來說，就算他們知道這是一個重要學科，卻不只被複雜的內容嚇到，也經常無法適應「定義-證明」的上課方式。為了克服這些學習上的困難，我們這堂課首先限制主題為跟 computer graphics 相關的線性代數，這樣不只可以讓學生用直覺的圖像來理解線性代數，而且 computer graphics 自己本身就是跟動畫、電玩有直接相關的領域。其次，我們限定這門課只處理 2D/3D (二維/三維)的線性代數，這樣並不會讓我們犧牲

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太多數學的本質，卻可以讓初學者避開抽象向量空間的符號。為證實這一點，我們可以往下面看到一個非常詳盡規劃的本課程進度說明。第一天我們將會對 2D/3D (二維/三維)的幾何物件做介紹，這是高中數學的複習。然後從第二天開始，我們就要設法改變這些幾何物件的形狀和移動他們的位置。在這裡，我們要操作的幾何物件，都是具有「直線」邊緣的，包括直線、平面、三角形、和金字塔角錐。而改變形狀和移動位置主要是靠下面六種線性變換: 伸縮、鏡射、旋轉、斜推、投影、反射。在第三天的課程，我們聚焦在一個非常特殊且重要的線性變換: 對稱矩陣。我們計劃講特徵值、特徵向量、正交變換、和對角化對稱矩陣。第四天，介紹重心坐標系，以及在 affine transformation 之下重心坐標系的座標變換。然後介紹如何將 2D/3D 幾何物件用數以千萬計的三角形好好連接起來，這是一些繪製 2D/3D 地形圖象的重要概念。最後一天，我們專講 2x2，3x3 矩陣的反矩陣演算法。包括高斯消去法，高斯消去法的幾何意義，以及 LU 分解法。

課程概述(英文)

Inevitably, linear algebra is a subject with a vast amount of details. For most students, even if they know this is an important subject, they are often scared, not only by the complicate materials involved, but also by the classical definition-proof style of approach. To overcome the difficulty, we first limit our focus only on computer graphics, a subject with many graphics on its own, which not only has the advantage to facilitate students in understanding linear algebra with intuitive pictures, but also it lends itself to the most current applications in animated movies and computer games. Secondly, we limit the course to only 2D and 3D spaces, with which we do not sacrifice by too much the mathematics essence, while avoiding abstract vector space notions for beginners. As one can see below, we provide a rather detailed syllabus which shows the exact contents to be covered by this course. We begin with a preliminary introduction which reviews basic geometry for 2D and 3D at nearly high school level. Then, starting from the second day, we are going to change the shape as well as to move location of those geometry objects. The geometry objects we will be mostly manipulating are those having a straight edges such as lines, plane, triangles, and pyramids. The action taken to move those objects are limited to Scaling; Reflections; Rotations; Shears; Projections; and Inverse linear transformations. In the third day, we focus on a very important class of linear transformations: symmetric matrices with which we plan to introduce eigenvalues, eigenvectors, diagonalization, and explain the related geometry concepts. In the 4th day, barycentric coordinates and 2D/3D triangulations: Describing how a geometric object can be approximated by millions of well-connected triangles. Finally, we devoted ourselves to solving 2x2 and 3x3 linear systems with an algorithmic approach: Gaussian eliminations and LU decompositions.

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課程進度

堂次	時間	進 度 說 明
6/29	**9:00-12:35**	**Preliminaries: Geometric objects in 2D and 3D with their representations.** **(i) Points and Vectors in 2D; Lengths; Angles; Orthogonal Projections; Inner Products; Linearly independence.** **(ii) Representing a Line: parametric form; implicit equation; explicit equation; Distance of a point to a line** **(iii) Barycentric Coordinate: convex combinations; Convex sets; 1-simplex (segment); 2-simplex (triangular); 3-simplex (Pyramid).** **(iv) From 2D to 3D: Cross Product; Lines; Planes; Distance of a point to a line and to a plane in 3D.**
6/30	**9:00-12:35**	**Changing shapes and moving geometric objects around: linear and affine transformations.** **(i) Linear transformation in matrix form: 2D and 3D cases.** **(ii) Six important linear transformations: Scaling; Reflections; Rotations; Shears; Projections; and Inverse linear transformations.** **(iii) Areas and determinants of matrices.** **(iv) Affine mappings and coordinate transformations** **(v) Mapping triangles to triangles** **(vi) (vi)Composing linear (affine) transformations.**
7/1	**9:00-12:35**	**Algebra and Geometry about Symmetric matrices.** **(i) Symmetric matrices (dimensionality and basis); Eigenvalues; Eigenvectors.** **(ii) Algebra: Diagonalizing a symmetric matrix. Orthogonal matrix.** **(iii) Geometry of symmetric matrices: Rotations; stretching; and undo Rotations.**
7/2	**9:00-12:35**	**Simplices and Conics.** **(i) Barycentric coordinates are affine invariant.** **(ii) Centroid of a simplex is also affine invariant.** **(iii) 2D and 3D triangulations: Describing a geometric object by millions of well-connected triangles.** **(iv) Lighting and shading in computer graphics.** **(v) Conic section for curves: parabola; ellipse; hyperbola in a general form and then transforming it back to its "standard" position.**
7/3	**9:00-12:40**	**Solving linear systems.** **(i) 2x2 cases: Gauss Eliminations (with a graphic illustration).** **(ii) 3x3 cases: LU decomposition with Forward and Backward Solver.**

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課程學習目標

1. 理解 2D/3D 重要幾何物件的座標、向量、參數表示法，以及數學特性。
2. 學習如何使用矩陣運算去改變幾何物件的形狀或移動其位置。
3. 清楚了解電腦圖像背後的線性代數思維、技術、以及與相關圖像應用上的連結。

課程的重要性、跨域性與時代性

Linear algebra has become a very important subject in today's computer and computing world. In particular, the user interface on the screen and the animated movies (games) are indeed driven by linear algebra and algorithms in their background. On the other hand, linear algebra itself is a fundamental subject in mathematics, which, together with calculus, serves as a cornerstone for nearly all areas of mathematics. This course links linear algebra with computer graphics; coordinates mathematics with applications; and eventually provides an interesting visual experience for students who want to know linear algebra with an intuitive sense.

其他備註

參考書目：

Joseph A. Gallian. Contemporary Abstract Algebra.