理　學　院

111 學年度第二學期模組化課程

圖解線性代數-理論與演練
Graphic Linear Algebra – Theory and Practice

授課教師	任職單位	畢業學校
許瑞麟	國立成功大學數學系	北卡羅來納州立大學

課程類別	學分數	選必修	開課人數	其他注意事項
Lecture + Recitation	1.5	選修	40	

先修課程或先備能力

無

課程難易度

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建議修課學生背景

適合各領域學生修習

教學方法

講授 65 %，討論 35%

這是 1.5 學分課程。每一天我們都會發一張當天上課的「學習單」，裡面的題目和內容都是最基礎、最重要、一定要會、也一定會考核的觀念。每天上課分成 3 個區段: 9:00 – 10:15 (第一節)；10:15 – 10:45(休息與學習單詢問)；10:45 – 12:00 (第二節)；12:00 – 13:00 (午休與學習單詢問，助教會在場)；13:00 – 14:15 (第三節)；14:15– 14:45 (休息與學習單詢問)；14:45 – 15:30 (考試)。

評量方式

問題考試 60%，期末報告 25%，發問問題 15%

補充說明：

每天 14:45-15:30 小考，共 5 次，考試內容為當天上課內容，學習單內容必考。考試方式為開放式引導式考試。學生可於考試時舉手發問，老師或助教會趨前給予適當提示並引導作答。不舉手發問視為自動放棄權利。每次小考佔 12%，五次共 60%。期末報告主要是撰寫課程 1000 字(以上)心得報告，佔 25%，內容必須包含「課程總結」、「課程內容自我整理筆記」、以及「學習心得」三部分。於課程結束後隔周按規定時間內，繳交 pdf 檔上傳至 NCKU Moodle system。發問問題佔 15%。學生可於上課發問問題；課間或課後向助教、老師發問問題；或與助教、老師約時間至其辦公室問問題。問題可以是課程相關，也可以是其他疑問的請教。發問問題將會有紀錄，於課程結束後結算並計算成績。

學習規範

無

課程概述

線性代數是一門研究「線性函數」的數學科目，這些函數的圖形，就是直線、平面或高維度空間的超平面(其圖像只能憑想像)。如果僅僅只是這樣，那麼線性代數就會很簡單，也不會有太多的用途，但是，為什麼大家都聽說，線性代數是一個具有海量內容的科目，非常有用卻也特別難學? 我想，

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「直線圖形」是一個兩面刃，它的確提供我們一個可視的幾何圖形來幫助我們認知，但卻同時限制住我們的想像力！平面的直線是 $R \to R$，空間中平面是$R^2 \to R$，超空間的超平面是$R^n \to R$，而數學線性代數這個學科卻是研究任何兩個向量空間 $U \to V$ 之間的線性變換。只要U 和 V 兩者的維度加起來超過 3，我們的眼睛就看不到了！而今天線性代數之所以有用，是因為

我們只要學會線性函數的機械式代數運算，不用看到圖形，就可以有效的處理我們想要知道的相關數學結果。但是，從圖形視覺過渡到抽象代數運算卻是需要經過特別的訓練。然而現今大部分線性代數的教學，卻是靠「定義-證明」的上課方式，硬碰硬的去攻這門困難的學科。這堂模組化課利用圖像方式來理解線性代數，因此限定在 2D/3D (二維/三維)的幾何物件之間的搬動。請注意，搬移前(in U)和搬移後(in V)的維度總和已經超過 3。這樣的限制，已經可以看到足夠多的數學的本質，卻可以讓初學者避開抽象向量空間的符號。第一天我們將會對 2D/3D (二維/三維)的幾何物件做介紹，這是高中數學的複習。然後從第二天開始，我們就要設法改變這些幾何物件的形狀和移動他們的位置。而改變形狀和移動位置主要是靠下面六種線性變換: 伸縮、鏡射、旋轉、斜推、投影、反射。在第三天的課程，我們聚焦在一個非常特殊且重要的線性變換: 對稱矩陣。我們計劃講特徵值、特徵向量、正交變換、和對角化對稱矩陣。第四天，介紹重心坐標系，以及在 affine transformation 之下重心坐標系的座標變換。最後一天，我們專講 2x2，3x3 矩陣的反矩陣演算法。包括高斯消去法，高斯消去法的幾何意義，以及 LU 分解法。

課程概述(英文)

Linear Algebra is a subject for linear mappings. It includes lines, planes, and hyperplanes (which cannot be visualized). Sounds easy, right? However, it is known to most people that linear algebra is a subject with a vast amount of details, very essential, but very difficult. The seeming contradiction arises from "visualization.'' It helps comprehend, but it also limits our imagination. A line is the graph for a function from $R \to R$, a plane is the graph for a function from $R^2 \to R$，a hyperplane demonstrates $R^n \to R$, while linear algebra is a subject for a mapping from any vector space U to another vector space V. As long as the dimension of U and V sums more than 3, we are unable to visualize it. It indicates that, for most of the time, we are incapable of seeing the graph of a linear mapping. Nevertheless, the power of today's linear algebra does not rely on truly visualizing the object, but on mechanic algebraic rules so that, by operating them, we know already the results of the mapping. Sounds great! But the important thing is that, it requires sufficient logic training in order to move from visualization to generating a correct understanding of what all those mathematical operations are doing. That is the utmost purpose of this modular course.

To achieve, we limit our focus on moving 2D/3D graphic pictures linearly. Notice that the total dimensionality before and after the transformation is already 4 or 6, which has the advantage to facilitate students in understanding linear algebra with intuitive pictures without sacrificing by too much the mathematics essence for beginners. As one can see below, we provide a rather detailed syllabus which shows the exact contents to be covered by this course. We begin with a preliminary introduction which reviews basic geometry for 2D and 3D at nearly high school level. Then, starting from the second day, we are going to change the shape as well as to move location of those geometry objects. The geometry objects we will be mostly manipulating are those having a straight edges such as lines, plane, triangles, and pyramids. The action taken to move those objects are limited to Scaling;

Reflections; Rotations; Shears; Projections; and Inverse linear transformations. In the third day, we focus on a very important class of linear transformations: symmetric matrices with which we plan to introduce eigenvalues, eigenvectors, diagonalization, and explain the related geometry concepts. In the 4th day, barycentric coordinates and 2D/3D triangulations. Finally, we devoted ourselves to solving 2x2 and 3x3 linear systems with an algorithmic approach: Gaussian eliminations and LU decompositions.

課程進度

堂次	時間	進 度 說 明
2023/1/9(一)	9:00-14:45	**Preliminaries:** (i) Local coordinate system and global coordinate system. (ii) Moving a box to a rectangle. (iii) Points and Vectors in 2D; Lengths; Angles; Orthogonal Projections; Inner Products; Linearly independence. (iv) Representing a Line: parametric form; implicit equation; explicit equation; Distance of a point to a line (v) Barycentric Coordinate: convex combinations; Convex sets; 1-simplex (segment); 2-simplex (triangular); 3-simplex (Pyramid). From 2D to 3D: Cross Product; Lines; Planes; Distance of a point to a line and to a plane in 3D.
	14:45-15:30	小考
2023/1/10(二)	9:00-14:45	Changing shapes and moving geometric objects around: linear and affine transformations. (i) Linear transformation in matrix form: 2D and 3D cases. (ii) Six important linear transformations in both 2D and 3D: Scaling; Reflections; Rotations; Shears; Projections (orthogonal and non-orthogonal projection); and Inverse linear transformations. (iii) Areas and determinants of 2x2 and 3x3 matrices . (iv) Affine mappings and coordinate transformations (v) Composing/decomposing linear (affine) transformations.
	14:45-15:30	小考
2023/1/11(三)	9:00-14:45	Algebra and Geometry about Symmetric matrices. (i) Symmetric matrices (dimensionality and basis); Eigenvalues; Eigenvectors. (ii) Diagonalizing a symmetric matrix. Orthogonal matrix. (iii) Geometry of symmetric matrices: Rotations; stretching; and undo Rotations.
	14:45-15:30	小考

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2023/1/12(四)	9:00-14:45	Applications, Simplices and Conics. (i) Applications to solving dynamic systems using eigenvalues and eigenvectors. (ii) Barycentric coordinates are affine invariant. (iii) Centroid of a simplex is also affine invariant. (iv) Conic section for curves: parabola; ellipse; hyperbola in a general form and then transforming it back to its “standard” position.
	14:45-15:30	小考
2023/1/13(五)	9:00-14:45	Solving linear systems. (i) 2x2 cases: Gauss Eliminations (with a graphic illustration). (ii) 3x3 cases: LU decomposition with Forward and Backward Solver.
	14:45-15:30	小考

課程學習目標

1. 理解 2D/3D 重要幾何物件的座標、向量、參數表示法，以及數學特性。
2. 學習如何使用矩陣運算去改變幾何物件的形狀或移動其位置。
3. 清楚了解電腦圖像背後的線性代數思維、技術、以及與相關圖像應用上的連結。

課程的重要性、跨域性與時代性

Linear algebra has become a very important subject in today’s computer and computing world. In particular, the user interface on the screen and the animated movies (games) are indeed driven by linear algebra and algorithms in their background. On the other hand, linear algebra itself is a fundamental subject in mathematics, which, together with calculus, serves as a cornerstone for nearly all areas of mathematics. This course approaches linear algebra graphically; incorporates mathematics with applications; and eventually provides an interesting visual experience for students who want to know linear algebra with an intuitive sense.

其他備註

參考書目：

Title: Practical Linear Algebra – A geometry toolbox
Authors: Gerald Farin and Dianne Hansford
Publisher: A K Peters, Ltd.