理　學　院

111 學年度第一學期模組化課程

平衡態的數學模式-從線性代數到微積分

Mathematical models for equilibrium – from linear algebra to calculus

授課教師	任職單位	畢業學校
許瑞麟	**國立成功大學數學系**	**北卡羅來納州立大學**

課程類別	學分數	選必修	開課人數	其他注意事項
Lecture + Recitation	**1.5**	**選修**	**30**	**無**

先修課程或先備能力

無

課程難易度

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建議修課學生背景

適合各領域學生修習

教學方法

講授 70%，演習與考試 30%

補充說明：

1. 每天下午助教先帶領作題目，下午最後 40 分鐘考試。考試內容以上課內容與演習課練習題為主。.

2. 若疫情嚴重，將改為線上授課，線上演習與線上考試。

評量方式

問題考試 50%，報告 40%，出席率 10%

補充說明：

每次小考佔 10%，五次共 50%。期末報告主要是撰寫課程(至少) 1000 字心得報告，內容必須包含「課程總結」、「數學結構與解法自我整合筆記」、以及「學習心得」三部分，於課程結束後隔周五，於 moodle 線上繳交。

學習規範

無

課程概述

本課程主要分兩部分。第一部分在數學模式構造方面，探討如何以矩陣和微積分來描述系統達到平衡態時的狀況。我們關注的系統包括: 彈簧系統、電路系統(離散); 彈性棒、二維流體力學(連續)。同時關注達到平衡態之前的簡諧運動，與最小位能的描述。第二部分是在數學解法上，講解線性系統的求解。在離散線性系統採用線性代數，連續線性系統則採用微積分，並且對照出兩者之間的橫向關聯性。傅立葉分析的整套求解方法會放在無窮維向量空間的座標化結構(帶有一組完備性的正交基底)去探討。課程最後一天的遞迴最小方差法也是平衡態的一個重要的應用。

課程概述(英文)

The course is arranged in two parts. In the first part, we first study how to model the equilibrium of states into a system of linear equations (in discrete systems, such as a line of springs or an electrical

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network); or into a (partial) differential equations (in continuous systems, such as an elastic bar or a 2D-fluid flow). In addition, modeling the simple harmonic motion with a view of minimum potential energy is also addressed. For the second part of the course, we are majorly concerned with the solution methods. For discrete linear systems, linear algebra, especially a matrix with its transpose is discussed. For a continuous linear systems, calculus should be applied to solve the system. Connection between linear algebra and calculus will be seen through the same algebraic structure. We focus on solving the Laplace equation (a continuous equilibrium equation) with a circle boundary condition via Fourier series, which will be put into the framework of an infinite dimensional vector space with a complete orthogonal system. The last day of the course will be designated to the method of recursive least square, which can be shown to be an important application of modeling the equilibrium of a system.

課程進度

日期	時間	進 度 說 明
8/22(一)	9:00 – 12:00	Equilibrium states for a line of springs. Diagram for the rank theorem in linear algebra. The Stiffness matrix.
	13:00 – 15:30	Recitation and quiz.
8/23(二)	9:00 – 12:00	Simple Harmonic Motion for a line of springs. Eigenvalue decomposition and solutions to ODE. Equilibrium states for a continuous bar. The minimum potential energy.
	13:00 – 15:30	Recitation and quiz.
8/24(三)	9:00 – 12:00	From mechanic to electrical network. Continuous flow on the plane. Connection to linear algebra and calculus: why a negative differential operator with a suitable boundary condition can be seen as the transpose of a matrix.
	13:00 – 15:30	Recitation and quiz.
8/25(四)	9:00 – 12:00	Solution to Laplace equation. Fourier series. An infinite dimension vector spaces and linear integral operators on it. This is another connection between linear algebra and calculus.
	13:00 – 15:30	Recitation and quiz.
8/26(五)	9:00 – 12:00	Recursive least square methods also share the same equilibrium structure.
	13:00 – 15:30	Recitation and quiz.

課程學習目標

1. **Learning how to model a physical equilibrium states into a system of linear equations or a Laplace equation.**
2. **Learning related linear algebra knowledge, including a matrix and its transpose; rank theorem; concept of null spaces; dual linear functional spaces.**
3. **Learning fundamental knowledge on Fourier analysis, including complex analysis, complete orthogonal systems, convergence issues.**

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4.Being able to piece together mathematical contents behind different applications. The unifying view will enable the students to face new applications in the future.

課程的重要性、跨域性與時代性

This course bridges between calculus and linear algebra; between mathematics and physics; and between mathematics and applied mathematics. It focuses on equilibrium equations for physical phenomena, from mechanics to electrical networks; and from discrete to continuous. Solution methods for solving the Laplace equation on the unit disc delivers a concise and complete introduction to the theory of Fourier series.

其他備註

參考書目：

Introduction to Applied Mathematics, by Gilbert Strange (MIT)